

1 - 5 Application of differential operators

Apply the given operator to the given functions. Show all steps in detail.

1. $D^2 + 2D$; $\text{Cosh}[2x]$, $e^{-x} + e^{2x}$, $\text{Cos}[x]$

```
ClearAll["Global`*"]
```

```
r[x_] := Cosh[2 x]
```

```
r''[x] + 2 r'[x]
```

```
4 Cosh[2 x] + 4 Sinh[2 x]
```

```
FullSimplify[%]
```

```
4 e2 x
```

```
s[x_] := e-x + e2 x
```

```
s''[x] + 2 s'[x]
```

```
e-x + 4 e2 x + 2 (-e-x + 2 e2 x)
```

```
Simplify[%]
```

```
e-x (-1 + 8 e3 x) (* text answer is -e-x+8e2x *)
```

```
u[x_] := Cos[x]
```

```
u''[x] + 2 u'[x]
```

```
-Cos[x] - 2 Sin[x]
```

The above answers agree with the text (except yellow).

3. $(D - 2I)^2$; e^{2x} , $x e^{-2x}$

```
ClearAll["Global`*"]
```

```
v[x_] := e2 x
```

```
v''[x] - 4 v'[x] + 4 v[x]
```

```
0
```

```
w[x_] := x e2 x
```

```
w''[x] - 4 w'[x] + 4 w[x]
```

```
4 e2 x + 8 e2 x x - 4 (e2 x + 2 e2 x x)
```

Simplify[%]

0

z[x_] := e^{-2 x}

z''[x] - 4 z'[x] + 4 z[x]

16 e^{-2 x}

The above answers agree with the text.

5. (D - 2 I)(D + 3 I); e^{2x}, x e^{2x}, e^{-3x}

ClearAll["Global`*"]

r[x_] := e^{2 x}

r''[x] + r'[x] - 6 r[x]

0

s[x_] := x e^{2 x}

s''[x] + s'[x] - 6 s[x]

5 e^{2 x}

u[x_] := e^{-3 x}

u''[x] + u'[x] - 6 u[x]

0

The above answers agree with the text.

6 - 12 General solution

Factor as in the text and solve.

7. (4 D² - I)y = 0

ClearAll["Global`*"]

r = 4 y''[x] - y[x] == 0

-y[x] + 4 y''[x] == 0

DSolve[r, y, x]

{ {y → Function[{x}, e^{x/2} C[1] + e^{-x/2} C[2]]} }

The above answer matches the text's.

$$9. (D^2 - 4.20 D + 4.41 I)y = 0$$

```
ClearAll["Global`*"]
```

$$s = y''[x] - 4.20 y'[x] + 4.41 y[x] == 0$$

```
DSolve[s, y, x]
```

$$4.41 y[x] - 4.2 y'[x] + y''[x] == 0$$

$$\left\{ \left\{ y \rightarrow \text{Function} \left[\{x\}, e^{2.1 x} C[1] + e^{2.1 x} x C[2] \right] \right\} \right\}$$

The above answer matches the text's.

$$11. (D^2 - 4.00 D + 3.84 I)y = 0$$

```
ClearAll["Global`*"]
```

$$u = y''[x] - 4.00 y'[x] + 3.84 y[x] == 0$$

```
DSolve[u, y, x]
```

$$3.84 y[x] - 4. y'[x] + y''[x] == 0$$

$$\left\{ \left\{ y \rightarrow \text{Function} \left[\{x\}, e^{1.6 x} C[1] + e^{2.4 x} C[2] \right] \right\} \right\}$$

The above answer matches the text's.